What you'll Learn About
- The Dot Product/Angle Between Vectors
- Projecting One Vector onto another/Work

Find the dot product of of u and v.

A) $\mathbf{u} = \langle -1,3 \rangle$, $\mathbf{v} = \langle 4,7 \rangle$

$u = \langle -1, 3 \rangle \quad v = \langle 4, 7 \rangle$

$u \cdot v = (-1)(4) + (3)(7)$

$= -4 + 21$

$u \cdot v = 17$

u dot v

1) multiply the x components
2) multiply the y components
3) Add the results

B) $\mathbf{u} = \langle -3,5 \rangle$, $\mathbf{v} = \langle -2,-6 \rangle$

$u \cdot v = (-3)(-2) + 5(-6)$

$= 6 + (-30)$

$= -24$

C) $(2\mathbf{i} - \mathbf{j}) \cdot (3\mathbf{i} - 5\mathbf{j})$

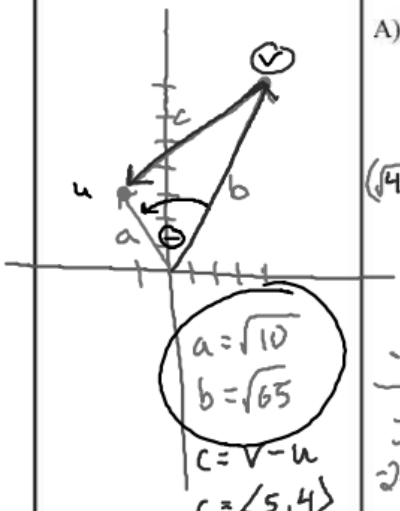
$\langle 2, -1 \rangle \cdot \langle 3, -5 \rangle = (2)(3) + (-1)(-5) = 6 + 5 = 11$

$i = \langle 1, 0 \rangle$

$j = \langle 0, 1 \rangle$

Find the angle between the two vectors.

A) $\mathbf{u} = \langle -1,3 \rangle$, $\mathbf{v} = \langle 4,7 \rangle$ → $u \cdot v = -4 + 21 = 17$



$a = \sqrt{10}$

$b = \sqrt{65}$

$c = v - u$

$c = \langle 5, 4 \rangle$

$|c| = \sqrt{41}$

$c^2 = a^2 + b^2 - 2ab\cos\theta$

$(\sqrt{41})^2 = \sqrt{10}^2 + \sqrt{65}^2 - 2(\sqrt{10}\sqrt{65})\cos\theta$

$41 = 10 + 65 - 2\sqrt{10}\sqrt{65}\cos\theta$

$41 = 75 - 2\sqrt{10}\sqrt{65}\cos\theta$

$-75 \quad -75$

$\frac{-34}{-2\sqrt{10}\sqrt{65}} = \frac{-2\sqrt{10}\sqrt{65}\cos\theta}{-2\sqrt{10}\sqrt{65}}$

$\frac{17}{\sqrt{10}\sqrt{65}} = \cos\theta$

$\theta = \cos^{-1}\left(\frac{17}{\sqrt{10}\sqrt{65}}\right)$

$\theta = 48.179$

Proof

A) $\mathbf{u} = \langle -1,3 \rangle$, $\mathbf{v} = \langle 4,7 \rangle$

$\theta = \cos^{-1}\left(\frac{u \cdot v}{|u||v|}\right)$

Find the angle between the two vectors.

B) $\mathbf{u} = \langle -3,5 \rangle$, $\mathbf{v} = \langle -2,-6 \rangle$

$\theta = \cos^{-1}\left(\frac{u \cdot v}{|v||v|}\right)$

$u \cdot v = (-3)(-2) + 5(-6) = -24$

$|u| = \sqrt{34}$

$|v| = \sqrt{40}$

$\theta = \cos^{-1}\left(\frac{-24}{\sqrt{34}\sqrt{40}}\right)$

Determine if the vectors are parallel, orthogonal, or neither.

A) $\mathbf{u} = \langle 2,3 \rangle$, $\mathbf{v} = \langle -6,4 \rangle$

B) $\mathbf{u} = \langle -3,5 \rangle$, $\mathbf{v} = \langle -2,-6 \rangle$

C) $\mathbf{u} = \langle -2,10 \rangle$, $\mathbf{v} = \langle -1,5 \rangle$

D) $\mathbf{u} = \langle -2,10 \rangle$, $\mathbf{v} = \langle 1,-5 \rangle$